Unified Engineering Spring **2008**

Signals and Systems Problems S5, S6, S7

Reading: O&W-6.5.2, 3.9 (continuous time systems), 4.1, Lectures S3 & S4

S5) A LTI system, with input *x(t)* and output *y(t),* is described by the following differential equation

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n\frac{dy(t)}{dt} + \omega_n^2 y(t) = \omega_n^2 x(t)$$

which is written in standard form for second order systems, where

ω_n = undamped natural frequency

ζ = damping ratio

This system has a frequency response given by

$$H(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2}$$

and the magnitude and angle of this function, for various values of the damping ratio, are depicted in Figure 6.23, on page 455 in the text.

Suppose that we have such a second order system as a piece of hardware and we know its undamped natural frequency (ω_n) quite accurately as

$$\omega_n = 1.0\ rad/\text{sec}$$

However, its damping ratio (ζ) is only known to be somewhere in the range $0.5 \leq \zeta < 1.0$.

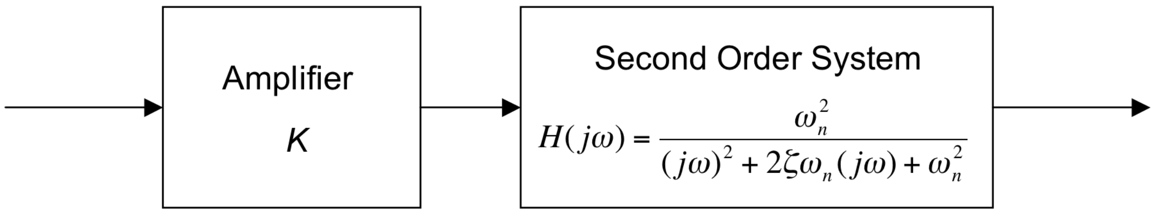
We wish to determine its damping ratio by testing the system using a square wave input. The square wave is to switch between 0.0 and +1.0 and its fundamental frequency is to be 1/3 the frequency of the system undamped natural frequency

i) What is the period of the square wave input?

ii) Determine the Fourier coefficients of the input for $k=0, \pm 1, \pm 2, \pm 3$
[Hint: refer to the square wave that we derived in class]

iii) What are the corresponding Fourier coefficients for the output if the system damping ratio is $\zeta = 0.7$?

S6) A bandpass filter is to be designed to pass signals at the frequency of 1.0 rad/sec and to filter out frequency content at frequencies above and below that frequency. In particular, at a frequency of 0.1 radians per second the filter must reduce the magnitude of any signal content by 40 Db. The filter is to consist of an amplifier followed by a second order system. A block diagram of the system is as follows:



The second order system undamped natural frequency (ω_n) is set at 1.0 radians per second.

i) Determine the damping ratio (ζ) of the second order system and the amplifier gain (*K)* so that the magnitude of the input/output frequency response satisfies the following requirements:

$$|K\,H(j\omega)| = 0.0\text{ d}B \quad \text{at } \omega = 1.0\text{ rad/sec}$$

$$|K\,H(j\omega)| = -40.0\text{ d}B \quad \text{at } \omega = 0.1\text{ rad/sec}$$

ii) What is the magnitude, in dB, of the bandpass filter frequency response at 10.0 rad/sec?

S7) Determine the Fourier transform for the following function of time

$$x(t) = \begin{cases} t+1 & -1 < t < 0 \\ 1-t & 0 < t < +1 \\ 0 & \text{elsewhere} \end{cases}$$